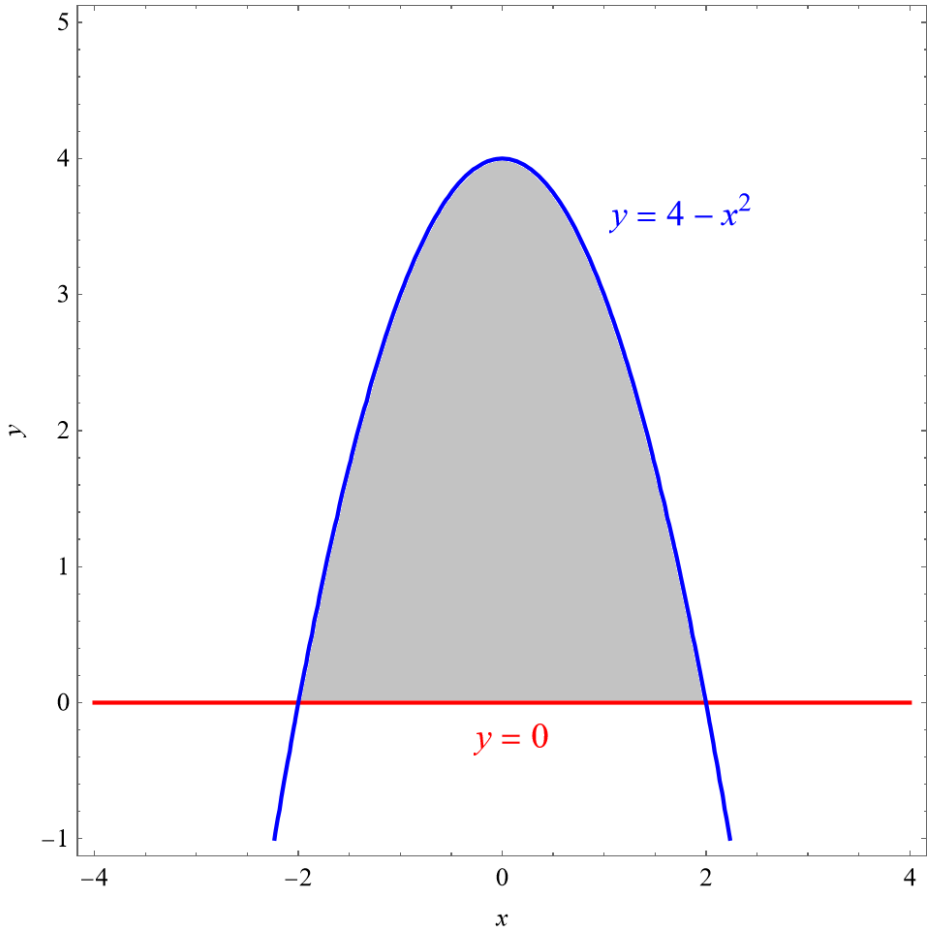
Exercise 47

Sketch the region enclosed by the given curves and calculate its area.

$$y = 4 - x^2, \quad y = 0$$

Solution

Start by drawing the given curves in the xy-plane and shading the area they enclose.



The shaded area is calculated by integrating the height $(4 - x^2 - 0)$ from the lowest value of x to the highest value of x that it occupies.

$$\text{Area} = \int_{x_{\min}}^{x_{\max}} \text{Height}\, dx = \int_{-2}^{2} (4 - x^2)\, dx$$

$$= \left(4x - \frac{x^3}{3}\right)\Bigg|_{-2}^{2}$$

$$= \left[4(2) - \frac{(2)^3}{3}\right] - \left[4(-2) - \frac{(-2)^3}{3}\right] = \frac{32}{3}$$